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Effect of Geometric Parameters on the Friction Factor in Periodically Constricted Tubes

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In a valuable contribution Payatakes et al. (1973) have solved the Navier-Stokes equations without neglecting any term, for a special case of periodically constricted tubes. Examples of their results have been given in two dimensionless diagrams (their Figures 8 and 9) which have been included here for reference as Figures 1 and 2. In both figures the friction factor of a uniform periodically constricted tube is plotted versus the Reynolds number. In Figure 1, the ratio $d_v^* = d_v/\tau$, where d_v is volumetric diameter and τ is length of a segment of a periodically constricted tube, was kept constant, equal to one; $\Delta r^* = (r_2 - r_1)/\tau$, where r_2 and r_1 are the maximum radius and the radius of the entrance constriction, respectively, was assigned different values, ranging from 0.5 to 0. In Figure 2, Δr^* was kept constant at 0.3, and d_v^* was assigned different values, ranging from 0.75 to ∞.

Payatakes et al. have stated that the conclusion reached by Batra (1969) and also reiterated by Batra, Fulford, and Dullien (1970), and by Dullien and Batra (1970), namely, that "The value of the friction factor increases with decreasing wave length to diameter ratio (< 0.5) by as much as 120% of the uniform tube value (wave amplitude to diameter ratios were in the range 0.13 to 0.25)" is incorrect. The criticism was based on an analysis of the above conclusion using the plots of Figure 2, notwithstanding the substantial difference between the geometry of the extensible flex-tubes used by Batra and that of their model. They also ignored the fact that Figure 2 does not apply to Batra's experiments in which Δr^* was not constant, but varied by as much as 27%. The reason for this variation is that it is not possible to vary $r_2 - r_1$ independently of τ in the case of extensible flex-tubes. Payatakes et al. were aware of these facts since they wrote "in the case of a flexible tube a decrease in τ/d_v is accompanied by a simultaneous increase in Δr^*".

Payatakes et al. allege also that in Batra's work "any change in the friction factor was attributed solely to the change in the ratio τ/d_v". It is unfortunate that these authors seem to have overlooked in their literature search the paper by Batra, Fulford, and Dullien (1970) where it has been demonstrated that in Batra's work the effect of τ/d_v on f_v far outweighed the effect of $(r_2 - r_1)/d_v$. This may be seen from Figures 3 and 4 with the accompanying tables, showing some of Batra's results. (We found it more convenient to use the dimensionless ratio $(r_2 - r_1)/d_v$ instead of $(r_2 - r_1)/\tau$.)

The data shows that both τ/d_v and $(r_2 - r_1)/d_v$ decreased whereas f_v increased. For obvious reasons a decrease in $(r_2 - r_1)/d_v$, that is, decrease in $(r_2 - r_1)$ for a fixed d_v, must cause a decrease in f_v, so that only the decrease in τ/d_v could have caused the observed increase in f_v.

The fact that decreasing τ for fixed values of d_v and $r_2 - r_1$, can indeed be expected to cause an increase in f_v may be understood if one first considers the limiting case of a uniform tube (one, without any constrictions) for which $\tau/d_v = \infty$, and then starts to introduce periodic irregularities into the tube walls to produce a culvert-like surface. It is obvious that, for a fixed d_v and $r_2 - r_1$, the friction factor f_v will tend to increase with decreasing wave length τ.

Hence, if f_v is regarded as a function of two variables

$$f_v = f_v[(r_2 - r_1)/d_v,\ \tau/d_v],$$

there can be little doubt that

$$\{\partial f_v/\partial[(r_2 - r_1)/d_v]\}_{\tau/d_v} > 0,$$

and

$$\{\partial f_v/\partial(\tau/d_v)\}_{(r_2-r_1)/d_v} < 0.$$

The first of these trends is also confirmed by the calculations of Payatakes et al. In Figure 1 $d_v^* = d_v/\tau = 1 =$ constant, implying that τ is constant for a fixed d_v.

Thus, increasing $(r_2 - r_1)/\tau$ implies increasing $(r_2 - r_1)/d_v$. The graphs in Figure 1 show that f_v is increasing with increasing $(r_2 - r_1)/\tau$ and, hence, increasing $(r_2 - r_1)/d_v$.

In Figure 2, however, $\Delta r^* = (r_2 - r_1)/\tau$ is kept constant and $d_v^* = d_v/\tau$ is varied, which amounts to varying d_v while keeping $r_2 - r_1$, and τ constant. This, of course, is the same thing as varying both $r_2 - r_1$, and τ in the same proportion while keeping d_v constant. The graphs in Figure 2 indicate an increase of f_v with decreasing d_v/τ, that is, with increasing τ and $r_2 - r_1$, with d_v kept constant. (Note that τ and $r_2 - r_1$ are always increased in the same proportion so as to keep the ratio $(r_2 - r_1)/\tau$ constant.) It is evident from the diagram that under these rather special conditions the effect of $r_2 - r_1$ (causing f_v to increase) outweighed that of τ (causing f_v to decrease). This behavior might appear to be contradicting the experimental results of Batra. These results, however, cannot be considered general for several obvious reasons. Because of the special way in which the effects of the two independent variables $r_2 - r_1$ and τ are combined, one cannot even conclude that the absolute value of $\partial f_v/\partial[(r_2 - r_1)/d_v]$ is greater than that of $\partial f_v/\partial(\tau/d_v)$, as the simultaneous changes in $(r_2 - r_1)/d_v$ and in τ/d_v are not in general of the same magnitude. The results would be much easier to interpret if in Figure 2 $(r_2 - r_1)/d_v$ had been kept constant and τ/d_v had been varied.

Payatakes et al., however, appear to have also performed calculations using some values of the parameters in Batra's experiments, the results of which were plotted in their Figure 10. It is evident from this diagram that their calculations have predicted decreasing f_v with increasing τ/d_v and $(r_2 - r_1)/d_v$ also for the specific conditions of some of Batra's experiments. This, of course, amounts to a direct conflict between the experimental measurements of Batra and the calculated results of Payatakes et al. Whether this basic disagreement is due to the different geometries of the flex-tubes and the model, or is caused by some flaw in the model or in the experimental work, cannot be decided at this time. It is a fact, however, that the great mass of data gathered by Batra on flextubes consistently gave the same dependence of f_v on τ and $r_2 - r_1$, for constant d_v.

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NOTATION

d_v = volumetric diameter of a uniform periodically constricted tube
d_v^* = dimensionless volumetric diameter
f_v = friction factor
r_1 = radius of constriction
r_2 = maximum radius
r_1^* = dimensionless radius of constriction (r_1/τ)
r_2^* = dimensionless maximum radius (r_2/τ)

Greek Letters

Δr = $r_2 - r_1$, amplitude
Δr^* = $r_2^* - r_1^*$, dimensionless amplitude
τ = length of segment of a periodically constricted tube

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Further Work on the Flow Through Periodically Constricted Tubes—A Reply

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In the preceding note (1973), Dullien and Azzam raised certain questions concerning the earlier work of Payatakes et al. (1973). In general, their major criticisms center around two issues: (1) the interpretation of the comparison between the experimental results of Batra (1969) and the numerical solution obtained by Payatakes et al. (1973), and (2) the correct assessment of the various geometry factors affecting the pressure drop (friction